Basic signal detection theory model

```
import os

if "KERAS_BACKEND" not in os.environ:
    # set this to "torch", "tensorflow", or "jax"
    os.environ["KERAS_BACKEND"] = "jax"

import matplotlib.pyplot as plt
import numpy as np

import keras
import bayesflow as bf

from scipy.stats import norm
```

In this example we will fit a basic signal detection theory model:

$$\begin{aligned} d &\sim \text{Normal}\left(0, \sqrt{2}\right) \\ c &\sim \text{Normal}\left(0, \sqrt{\frac{1}{2}}\right) \\ \theta_h &\leftarrow \Phi\left(\frac{d}{2} - c\right) \\ \theta_f &\leftarrow \Phi\left(-\frac{d}{2} - c\right) \\ h &\sim \text{Binomial}(\theta_h, n_{\text{signal}}) \\ f &\sim \text{Binomial}(\theta_f, n_{\text{noise}}), \end{aligned} \tag{1}$$

where h, f are the number of hit rates and false alarms, out of n_{signal} and n_{noise} trials, respectively. We will estimate the distriminability d and bias c. During the inference stage we will be also interested in the implied hit rate θ_h and false alarm rate θ_f.

Simulator

Setting up the simulator is straightforward. We wish to be able to make inferences for different numbers of trials, and so we will vary that as a context variable.

```python
def context():
    return dict(n = np.random.randint(low=5, high=120, size=2))

def prior():
    return dict(
        d = np.random.normal(0, np.sqrt(2)),
        c = np.random.normal(0, np.sqrt(1/2))
    )

def likelihood(d, c, n):
    # compute hit rate and false alarm rate:
    theta = norm.cdf([0.5 * d - c, - 0.5 * d - c])
    # simulate hits and false alarms
    x = np.random.binomial(n=n, p=theta, size=2)

    return dict(x=x)

simulator = bf.make_simulator([context, prior, likelihood])
```

Approximator

We will use a `BasicWorkflow` object to wrap up all necessary networks.

```python
workflow = bf.BasicWorkflow(
    simulator=simulator,
    inference_network=bf.networks.CouplingFlow(),
    inference_variables=["d", "c"],
    inference_conditions=["x", "n"]
)
```

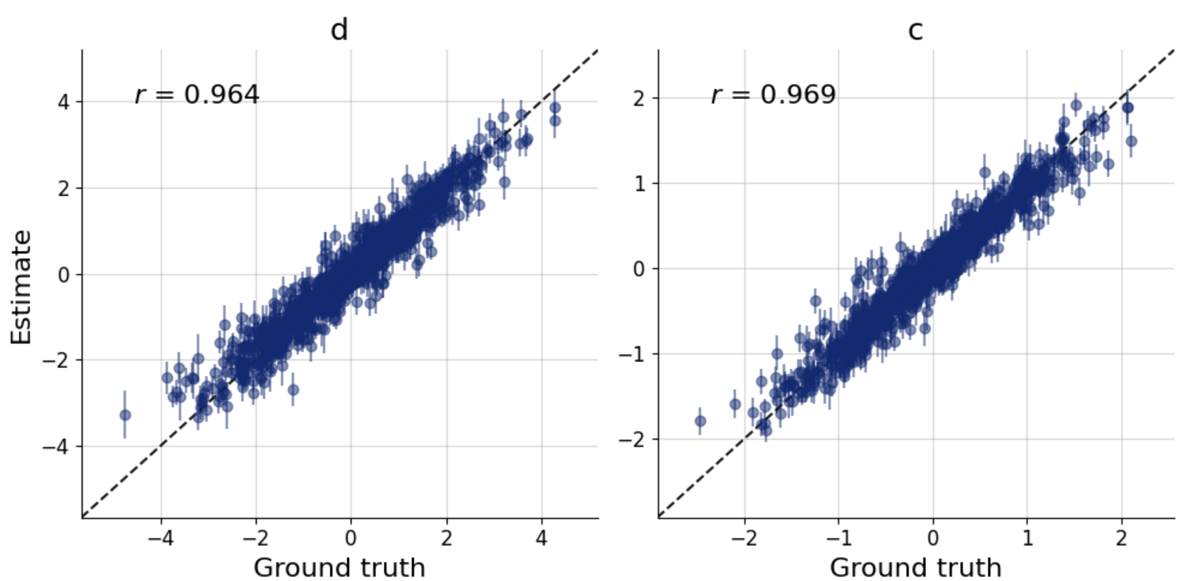
Training

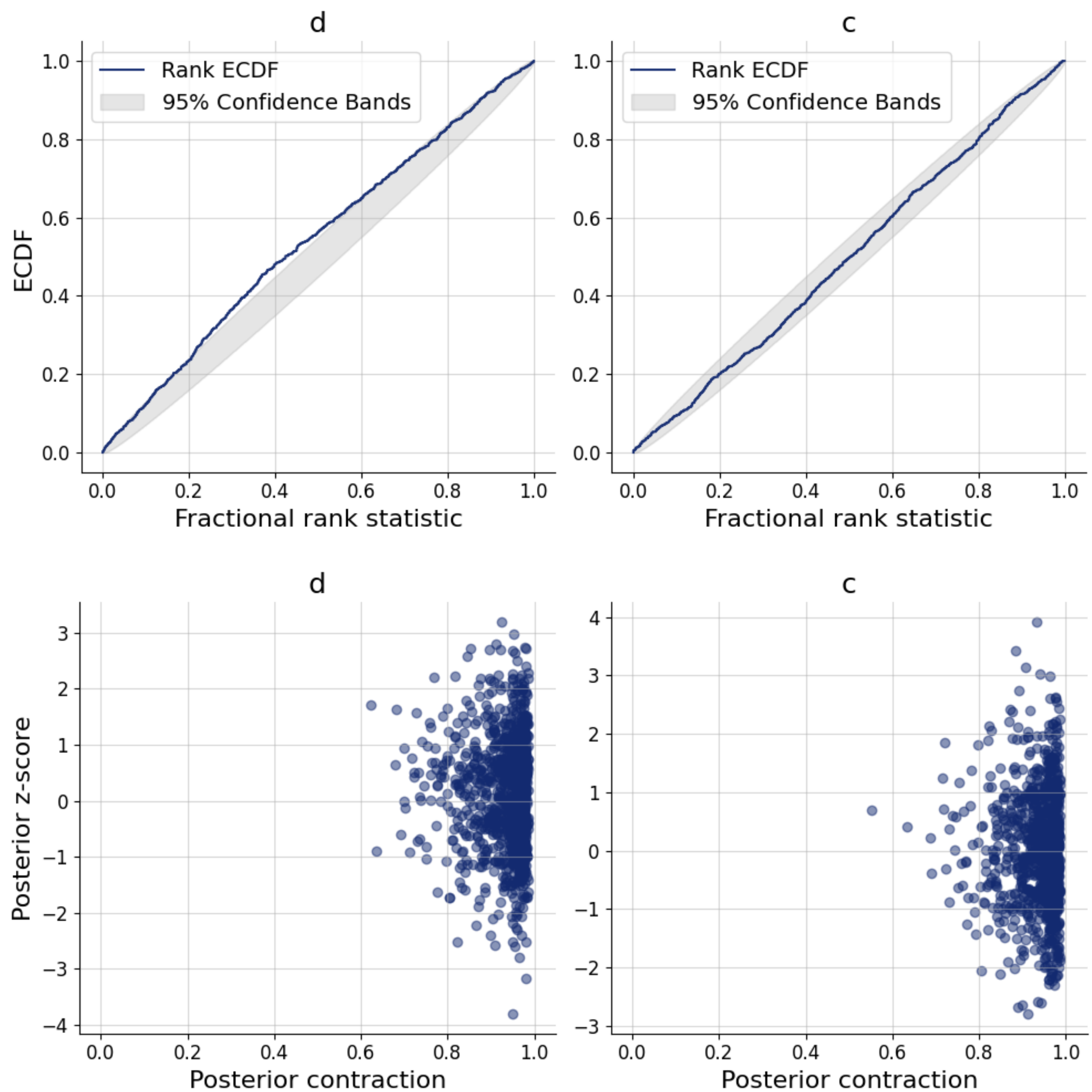
```python
history = workflow.fit_online(epochs=50, num_batches_per_epoch=200, batch_size=256)
```

Validation

```
test_data = simulator.sample(1000)
```

```
{'losses': <Figure size 1600x400 with 1 Axes>,
 'recovery': <Figure size 1000x500 with 2 Axes>,
 'calibration_ecdf': <Figure size 1000x500 with 2 Axes>,
 'z_score_contraction': <Figure size 1000x500 with 2 Axes>}
```






```
figs = workflow.plot_default_diagnostics(test_data=test_data, num_samples=500)
```

Inference

We are given three data sets (Lee & Wagenmakers, 2013):

1. $h = 70$, $f = 50$, $n_{\text{signal}} = n_{\text{noise}} = 100$
2. $h = 7$, $f = 5$, $n_{\text{signal}} = n_{\text{noise}} = 10$

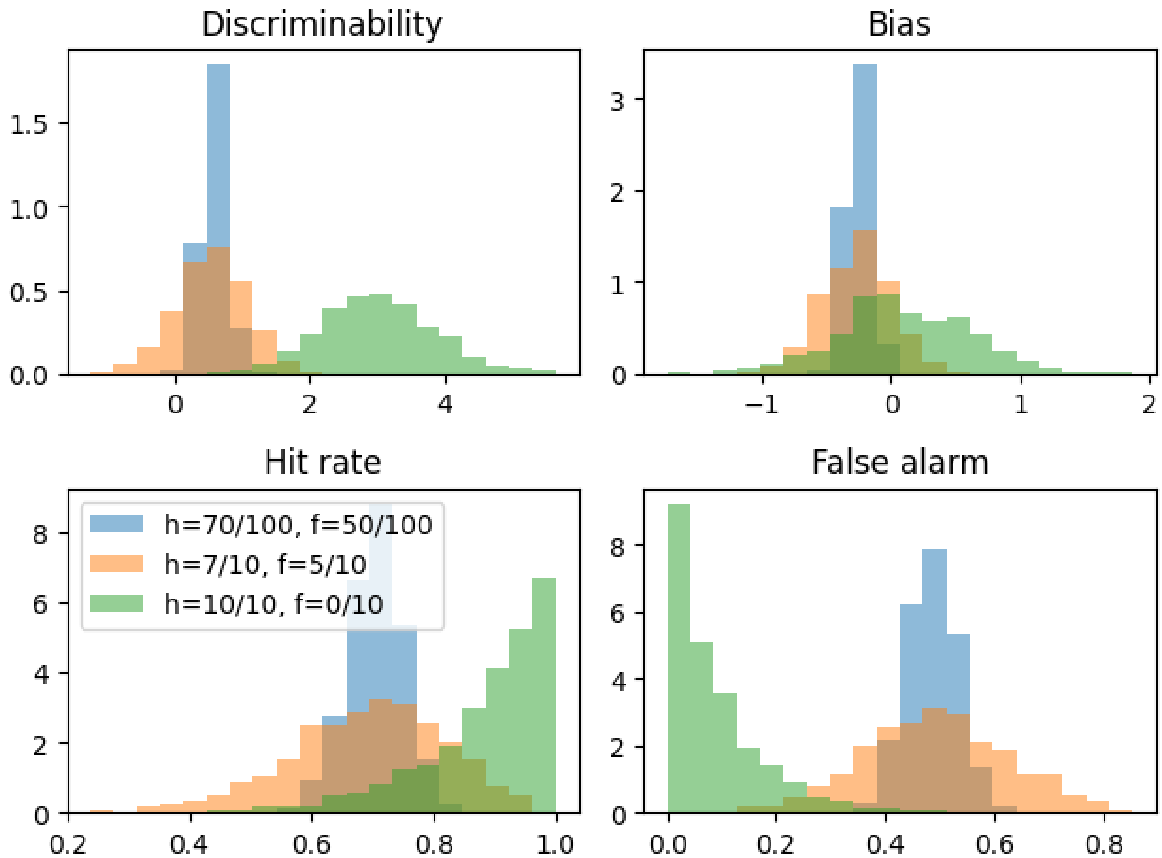
3. $h = 10$, $f = 0$, $n_{\text{signal}} = n_{\text{noise}} = 10$.

We can leverage the fact that we can make inferences for all three datasets at the same time, we will get an array of the posterior samples of d and c for each of the datasets.

```
inference_data = dict(
    x = np.array([
        [70, 50],
        [7, 5],
        [10, 0]
    ]),
    n = np.array([
        [100, 100],
        [10, 10],
        [10, 10]
    ])
)

posterior_samples = workflow.sample(conditions=inference_data, num_samples=1000)
```

```
def generated_quantities(c, d):
    return {
        "Discriminability": d,
        "Bias": c,
        "Hit rate": norm.cdf(0.5 * d - c),
        "False alarm": norm.cdf(-0.5 * d - c)
    }

posterior_samples = generated_quantities(**posterior_samples)
```

```
fig, axs = plt.subplots(2, 2)
axs=axs.flatten()

for index, (key, value) in enumerate(posterior_samples.items()):
    bins = np.histogram_bin_edges(value.ravel(), bins=20)
    for data_set in range(inference_data['x'].shape[0]):
        axs[index].hist(value[data_set].flatten(), alpha=0.5, density=True, bins=bins)
        axs[index].set_title(key)
axs[2].legend(['h=70/100, f=50/100', 'h=7/10, f=5/10', 'h=10/10, f=0/10'])
fig.tight_layout()
```



Lee, M. D., & Wagenmakers, E.-J. (2013). *Bayesian Cognitive Modeling: A Practical Course.* Cambridge University Press.